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Concepts as Modalities in Description Logics

Lukas Grätz

Institut für Philosophie, Universität Leipzig, Germany
lukas.graetz@studserv.uni-leipzig.de

Abstract

Motivated by the colloquial language term of a "glass gummy bear", an additional type of concept composition for description logics is suggested. This composition type is then axiomatically formalized and called concept generalization. Consistency of the formalization is checked. By proving axiom K and Gödel rule, it is shown that this logic is in fact a multi-modal logic. Concepts could be both modal operators and predicate symbols. A Kripke semantics is presented (the adequacy is future work). In this semantics, the TBox axioms hold for any view, assertions in the ABox hold for the natural view (a selected world in the Kripke structure) only. The relationship to other formalisms is outlined. Further examples are discussed at the end.

1 Set-Theoretic Semantics?

Several examples in language inspire us to invent a new type of concept composition for description logics (DL). Examples include a "stone lion", a "fake gun", "the biggest city", which is a composition of "city" and "the biggest", or "glass gummy bear", which composes "glass" and "gummy bear". A linguistic perspective can be found in [3]. It turns out that this concept composition cannot be defined within DL languages like $\mathcal{ALC}$. The other insight is that this composition introduces modal operators $\langle \mathit{TheBiggest} \rangle$ or $\langle \mathit{glass} \rangle$. To make it more clear that we are dealing with a binary operator I am using the notion $C \multimap D$ in contrast to $\langle C \rangle D$ used by [2].

Klarman [2] uses a set-theoretic semantic definition to formalize this type of concept composition. In his logic the following equivalence can be proved for all concepts C, D:

$$\langle D \rangle C \sqcap \langle D \rangle \neg C \equiv \bot \tag{1}$$

In the alternative notation:

$$D \multimap C \sqcap D \multimap \neg C \equiv \bot \tag{2}$$

The modal logic equivalent $\Diamond p \wedge \Diamond \neg p$ is contingent (and not a contradiction) in normal modal logics. The athletic modality "it is possible that p and it is possible that not p" should be represented and must not be a contradiction as we can see in propositions like "it is possible that the poster gets accepted and it is possible that the poster will be rejected".

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What about the new concept composition operator? Consider a knowledge base with a glass gummy bear:

TBox

$$\texttt{gummy\_bear} \sqsubseteq \texttt{edible} \quad (3)$$
$$\texttt{glass} \sqsubseteq \neg\texttt{edible} \quad (4)$$

ABox

$$\texttt{glass} \multimap \texttt{gummy\_bear}(a) \quad (5)$$

TBox axioms apply to any possible world, whereas the ABox should hold for a selected world only. We can derive by 3:

$$\texttt{glass} \multimap \texttt{edible}(a) \quad (6)$$

$\texttt{glass}(a)$ should follow from the ABox (see Section 2). Hence we can derive:

$$\neg\texttt{edible}(a) \quad (7)$$

A transition from $\texttt{glass} \sqcap \neg\texttt{edible}$ "glass and not edible" to $\texttt{glass} \multimap \neg\texttt{edible}$ "glass model of something not edible" is problematic. But why should it be contradictory to assume that a is also a glass model of something not edible?

The concept of uncertainty $\Diamond$ makes it even more clear: Even if we are not sure whether b is a gummy bear or not (perhaps because we only saw b from distance) we could add $\Diamond \multimap \texttt{gummy\_bear}$ to the ABox – and also $\Diamond \multimap \neg\texttt{gummy\_bear}$.

2 Constraints

The following constraints should hold for the $\multimap$ operator and for all concepts C, D, E (you could call them axioms instead, if the list would be complete):

$C \multimap (D \sqcup E) \sqsubseteq C \multimap D \sqcup C \multimap E$	Union	(8)
$C \multimap \top \equiv C$	Universality	(9)
$C \multimap \bot \equiv \bot$	Inconsistent Concept	(10)
$\top \multimap C \equiv C$	Realness	(11)
$\dfrac{D \sqsubseteq E}{C \multimap D \sqsubseteq C \multimap E}$	Generalization Rule	(12)

Note that the generalization symbol $\multimap$ has the strongest bond after $\neg$. Most of the constraints are easily defended:

(8)	A stone tiger-or-leopard is a stone tiger or a stone leopard.
(9)+(12)	A special C should be a C: $C \multimap D \sqsubseteq C \multimap \top \equiv C$
(11)	A real C should be a C.
(12)	The TBox should hold for any accessible view.

Axiom K and Gödel rule are provable using (8)+(10)+(12), hence any logic satisfying the constraints is a multi-modal logic.

Theorem 2.1 (Consistency)**.** *The extended logic is consistent, which means that $\bot \equiv \top$ does not follow from the constraints. Furthermore, $C \multimap D$ is satisfiable.*

Proof. We can transform the extension to $\mathcal{ALC}$ by replacing the $\multimap$ with $\sqcap$. The transformed constraints can be proved in $\mathcal{ALC}$. The transformed generalization rule is provable. Note that this transformation does not preserve the semantics! □

3 Formalization by Embedding in FOL

$\mathcal{ALC}$ can be embedded into first order logic (FOL) using a translation function $\pi_x : \mathcal{L}_{\mathcal{ALC}} \to \mathcal{L}_{FOL}$, which converts $\mathcal{ALC}$ formulas into FOL with one free variable x (see [4, 1])[1] Slightly modifying π gives us the desired properties (for C, D concepts and atomic concepts A)

$$
\begin{aligned}
{}^{v_1}\pi_x^{v_2} : \mathcal{L}_{\mathcal{ALC}}^{\multimap} &\to \mathcal{L}_{FOL} \\
C \sqcup D &\mapsto {}^{v_1}\pi_x^{v_2}(C) \ \vee \ {}^{v_1}\pi_x^{v_2}(D) && \text{(13a)} \\
\neg C &\mapsto v_1 = v_2 \ \wedge \ \neg \ {}^{v_1}\pi_x^{\bullet}(C) && \text{(13b)} \\
C \sqcap D &\mapsto {}^{v_1}\pi_x^{\bullet}(C) \wedge {}^{v_1}\pi_x^{\bullet}(D) \ \wedge \ \big({}^{v_1}\pi_x^{v_2}(C) \ \vee \ {}^{v_1}\pi_x^{v_2}(D)\big) && \text{(13c)} \\
C \multimap D &\mapsto \exists v_3 \ \big({}^{v_1}\pi_x^{v_3}(C) \ \wedge \ {}^{v_3}\pi_x^{v_2}(D)\big) && \text{(13d)} \\
\forall r.\ C &\mapsto \forall y \ \big(p_r(x,y) \to \ {}^{v_1}\pi_y^{v_2}(C)\big) && \text{(13e)} \\
\exists r.\ C &\mapsto \exists y \ \big(p_r(x,y) \ \wedge \ {}^{v_1}\pi_y^{v_2}(C)\big) && \text{(13f)} \\
A &\mapsto q_A(v_1, v_2) \ \wedge \ e_{v_2}(x) && \text{(13g)}
\end{aligned}
$$

where q_A, p_r and e_v are FOL-predicates ($p_\top(x,y) :\leftrightarrow x = y$, $p_\bot(x,y) :\leftrightarrow \bot$ and $e_{v*}(x) :\leftrightarrow x \in \Delta$ are predefined predicates). Furthermore, ${}^{v}\pi_x^{\bullet}(C) \ := \ \exists \tilde{v} \ {}^{v}\pi_x^{\tilde{v}}(C)$ is an abbreviation.

You may detect two distinct Kripke-Structures: First we have the views $v \in V$, which are accessible by the relations q_C for every concept C. Second, we have the set of individuals Δ, which are accessible by the relations p_r for every role r. The difference comes with the evaluation of a TBox $\mathcal{T} = \{C_i \sqsubseteq D_i | 1 \leq i \leq n\}$ defined as

$$\pi(\mathcal{T}) \ := \ \forall v \ \forall x \ \bigwedge_{i=1}^{n} \ {}^{v}\pi_x^{\bullet}(C) \to \ {}^{v}\pi_x^{\bullet}(D) \tag{14}$$

and ABox assertions $C(a)$:

$$\pi(C(a)) \ := \ {}^{v*}\pi_a^{\bullet}(C) \tag{15}$$

The TBox holds for every view $v \in V$, the ABox only for a designated view v^*. Without any $\multimap$ in ABox and TBox, only this designated view matters for the TBox to evaluate the ABox. In that case, other views only occur in

$$\exists \tilde{v} \ q_A(v^*, \tilde{v}) \ \wedge \ e_{v*}(x) \tag{16}$$

[1] For each transformation rule, two variants are needed for the permutations of the two variables x and y. In the modified embedding presented here, the transformation rules for the permutations ${}^{v_2}\pi_x^{v_1}$, ${}^{v_1}\pi_y^{v_2}$ and ${}^{v_2}\pi_y^{v_1}$ can be defined likewise.

as obtained from (13g). After defining $A(x) :\leftrightarrow$ (16), the embedding function ${}^{v*}\pi_x^{\bullet}$ turns out to be equivalent to the original pure $\mathcal{ALC}$ embedding π_x as defined in [4].

The constraints of the previous section are satisfied, hence the logic is sound (the proofs are straightforward but would exceed this paper).

4 Further Examples

First, a closer look at the biggest city as described in [2]. "Biggest city" turns out to be a relative term, since we distinguish between the biggest city in Asia $Asian\_City \multimap The\_Biggest$, the biggest city in Europe and the biggest city on earth. But how about an alternative formalization using conventional DL? It turns out that all we need would be a $Greater\_Than$ relation between cities. For instance, the biggest city in Asia could be defined as:

$$Biggest\_Asian\_City \ \equiv\ Asian\_City \sqcap \neg\exists Greater\_Than.\ Asian\_City \tag{17}$$

Let us try more promising examples: Another common relative terminology is the word "normal". A prominent example of an inconsistent knowledge base is that penguins cannot fly, whereas birds can fly. This could be solved by saying that normal birds (like pidgins) fly – penguins are birds but not normal.

$$bird \multimap normal \sqsubseteq flies \tag{18}$$

$$pidgin \sqsubseteq bird \multimap normal \tag{19}$$

$$penguin \sqsubseteq bird \tag{20}$$

The next example deals with melted things, like melted ice cream, melted water ice or melted chocolate. Melted ice cream is something that you would not call ice cream, because it is no longer creamy. Melted water ice, however, is still some aggregate form of water and can become ice again. Melted chocolate remains tasty but has lost its original form forever. The TBox could be formalized as follows:

$$melted \multimap ice\_cream \sqsubseteq \neg ice\_cream \tag{21}$$

$$melted \multimap water\_ice \sqsubseteq water \tag{22}$$

$$melted \multimap chocolate \sqsubseteq chocolate \tag{23}$$

An alternative formalization of "melted" by using a time logic is conceivable, which would require another extension of DL.

Last but not least, a vegetarian burger $\texttt{veggi} \multimap \texttt{burger}(a)$.

$$\texttt{burger} \sqsubseteq \exists\texttt{Ingredient.}\ \texttt{meat} \tag{24}$$

$$\texttt{burger} \sqsubseteq \exists\texttt{Ingredient.}\ \texttt{bread} \tag{25}$$

$$\texttt{veggi} \sqsubseteq \forall\texttt{Ingredient.}\ \texttt{veggi} \tag{26}$$

$$\texttt{veggi} \sqcap \texttt{meat} \equiv \bot \tag{27}$$

$$\texttt{veggi} \multimap \texttt{bread} \sqsubseteq \texttt{bread} \tag{28}$$

What kind of meat does a veggi burger contain? To answer this, we use the FOL embedding:

$${}^{v*}\pi_a^{\bullet}(\texttt{veggi} \multimap \texttt{burger}) \tag{29}$$

$$=_{def}\ \exists v_1\ \left({}^{v*}\pi_a^{v_1}(\texttt{veggi})\ \wedge\ {}^{v_1}\pi_a^{\bullet}(\texttt{burger})\right) \tag{30}$$

Axiom (24) yields:

$$\Longrightarrow \exists v_1 \left({}^{v*}\pi_a^{v_1}(\texttt{veggi}) \wedge {}^{v_1}\pi_a^{\bullet}(\exists \texttt{Ingr.}\ \texttt{meat})\right) \tag{31}$$

$$=_{def} \exists v_1 \left({}^{v*}\pi_a^{v_1}(\texttt{veggi}) \wedge \exists y\ [p_{\texttt{Ingr}}(a,y) \wedge {}^{v_1}\pi_y^{\bullet}(\texttt{meat})]\right) \tag{32}$$

$$\Longleftrightarrow \exists y\ [p_{\texttt{Ingr}}(a,y) \wedge \exists v_1 \left({}^{v*}\pi_a^{v_1}(\texttt{veggi}) \wedge {}^{v_1}\pi_y^{\bullet}(\texttt{meat})\right)] \tag{33}$$

Axiom (26) yields ${}^{v*}\pi_y^{v_1}(\texttt{veggi})$ that y is veggi too:

$$\Longrightarrow \exists y\ [p_{\texttt{Ingr}}(a,y) \wedge \exists v_1 \left({}^{v*}\pi_y^{v_1}(\texttt{veggi}) \wedge {}^{v_1}\pi_y^{\bullet}(\texttt{meat})\right)] \tag{34}$$

$$=_{def} \exists y\ [p_{\texttt{Ingr}}(a,y) \wedge {}^{v*}\pi_y^{\bullet}(\texttt{veggi} \multimap \texttt{meat})] \tag{35}$$

$$=_{def} {}^{v*}\pi_a^{\bullet}(\exists \texttt{Ingr.}\ \texttt{veggi} \multimap \texttt{meat}) \tag{36}$$

Hence a veggi burger contains "veggi meat" (according to the TBox). It is easy to show that "veggi meat" cannot be "meat" using (27). Likewise, a veggi burger contains "veggi bread" (which is real bread).

In conclusion, the DL extension is more expressive then basic DL, the veggi burger is a nice example. The scope of the approach is not yet determined. Although inspired by language, it has a clear semantics and no vagueness or ambiguity. This is a great advantage of logical formalism over natural language, if it is not the main goal to formalize natural language.

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